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A Possible Cepheid-Like Luminosity Estimator for the Long Gamma-Ray Bursts

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ABSTRACT

We present a possible Cepheid-like luminosity estimator for long gamma-ray bursts based on the variability of their light curves. To construct the luminosity estimator, we use *CGRO*/BATSE data for 13 bursts, *Wind*/KONUS data for 5 bursts, *Ulysses*/GRB data for 1 burst, and NEAR/XGRS data for 1 burst. Spectroscopic redshifts, peak fluxes, and high resolution light curves are available for 11 of these bursts; partial information is available for the remaining 9 bursts. We find that the isotropic-equivalent luminosities L of these bursts positively correlate with a rigorously-constructed measure V of the variability of their light curves. We fit a model to these data that accommodates both intrinsic scatter (statistical variance) and extrinsic scatter (sample variance). If one excludes GRB 980425 from the fit on the grounds that its association with SN 1998bw at a redshift of $z = 0.0085$ is not secure, the luminosity estimator spans ≈ 2.5 orders of magnitude in L , and the slope of the correlation between L and V is positive with a probability of $1 - 6.4 \times 10^{-4}$ (3.4σ). Although GRB 980425 is excluded from this fit, its L and V values are consistent with the fitted model, which suggests that GRB 980425 may well be associated with SN 1998bw, and that GRB 980425 and the cosmological bursts may share a common physical origin. If one includes GRB 980425 in the fit, the luminosity estimator spans ≈ 6.3 orders of magnitude in L , and the slope of the correlation is positive with a probability of $1 - 2.1 \times 10^{-6}$ (4.7σ). In either case, the luminosity estimator yields best-estimate luminosities that are accurate to a factor of ≈ 5 , or best-estimate luminosity distances that are accurate to a factor of ≈ 2 . Independently of whether or not GRB 980425 should be included in the fit, its light curve is unique in that it is

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much less variable than the other ≈ 16 light curves of bursts in our sample for which the signal-to-noise is reasonably good.

Subject headings: gamma-rays: bursts

1. Introduction

Since gamma-ray bursts (GRBs) were first discovered (Klebesadel, Strong, & Olson 1973), thousands of bursts have been detected by a wide variety of instruments, most notably, the Burst and Transient Source Experiment (BATSE) on the *Compton Gamma-Ray Observatory* (*CGRO*), which will have detected about 2700 bursts by the end of *CGRO*’s more than 9 year mission in 2000 June (see, e.g., Paciesas et al. 1999). However, the distance scale of the bursts remained uncertain until 1997, when BeppoSAX began localizing long bursts to a few arcminutes on the sky, and distributing the locations to observers within hours of the bursts. This led to the discovery of X-ray (Costa et al. 1997), optical (van Paradijs et al. 1997), and radio (Frail et al. 1997) afterglows, as well as host galaxies (Sahu et al. 1997). Subsequent observations led to the spectroscopic determination of burst redshifts, using absorption lines in the spectra of the afterglows (see, e.g., Metzger et al. 1997), and emission lines in the spectra of the host galaxies (see, e.g., Kulkarni et al. 1998a). To date, redshifts have been measured for 13 bursts.

Using a sample of 7 BATSE bursts for which spectroscopic redshifts, peak fluxes, and high resolution light curves were available, Ramirez-Ruiz & Fenimore (1999) have suggested that more luminous bursts are also more variable, and consequently that variability might be used as a luminosity estimator.⁶ In this paper, we present a possible luminosity estimator, the construction of which was motivated by the work of Ramirez-Ruiz & Fenimore (1999) and Fenimore & Ramirez-Ruiz (2000). We term the luminosity estimator “Cepheid-like” in that it can be used to infer burst luminosities, and hence luminosity distances, from their light curves alone.

We rigorously construct our measure V of the variability of a burst light curve in §2. Qualitatively, V is computed by taking the difference of the light curve and a smoothed version of the light curve, squaring this difference, summing the squared difference over time intervals, and appropriately normalizing the result. Our variability measure differs from that of Fenimore & Ramirez-Ruiz (2000) in three important ways: (1) we define the timescale over which the light curve is smoothed differently; (2) we subtract out the contribution to the variability due to Poisson noise differently; and (3) we combine variability measurements of light curves acquired in different energy bands into a single measurement of a burst’s variability differently.

⁶Stern, Poutanen, & Svensson (1999) also have proposed a trend between luminosity and variability, using 1310 BATSE bursts and their peak fluxes. Norris, Marani & Bonnell (2000) have proposed a trend between luminosity and spectral lags, using the same 7 bursts as Ramirez-Ruiz & Fenimore (1999) and their redshifts.

We find that only smoothing timescales that are proportional to burst duration appear to lead to significant correlations between L and V ; in particular, smoothing timescales that are a fixed duration in the source frame do not. We take the smoothing timescale of a burst light curve, acquired in observer-frame energy band E , to be the time spanned by the brightest $100f\%$ of the total counts above background, where f is a number between 0 and 1 that we fix using the data in §4. We schematically illustrate this measure of duration in Figures 1 and 2. In Figure 1, the 50% smoothing timescale of the light curve is given by $T_{f=0.5}^E = T_1 + T_2$, and the 90% smoothing timescale of the light curve is given by $T_{f=0.9}^E = T_3$; in Figure 2, the 50% smoothing timescale of the light curve is given by $T_{f=0.5}^E = T_4$, and the 90% smoothing timescale of the light curve is given by $T_{f=0.9}^E = T_5 + T_6$. We have chosen this measure of the smoothing timescale over others because it is robust: small variations in either the light curve or the value of f result in only small variations in the value of the smoothing timescale. This is not the case with measures like T_{50} and T_{90} (see, e.g., Paciesas et al. 1999).

We present our measure of the isotropic-equivalent luminosity L of a burst in §3. In §4, we expand the original Ramirez-Ruiz & Fenimore (1999) sample of 7 bursts to include a total of 20 bursts, including 13 BATSE bursts, 5 *Wind*/KONUS bursts, 1 *Ulysses*/GRB burst, and 1 NEAR/XGRS burst. Spectroscopic redshifts, peak fluxes, and high resolution light curves are available for 11 of these bursts; partial information is available for the remaining 9 bursts.

Also in §4, we construct our luminosity estimator, which differs from that of Fenimore & Ramirez-Ruiz (2000) in two important ways: (1) applying the Bayesian inference formalism developed by Reichart (2000), we fit a model to the data that accommodates both intrinsic scatter (statistical variance) in two dimensions and extrinsic scatter (sample variance) in two dimensions; and (2) again applying the Bayesian inference formalism developed by Reichart (2000), we determine not only the best estimate for L as a function of V , but also the uncertainty in L as a function of V , as well as approximate these functions with analytic expressions. As a result of the differences in our variability and luminosity measures, we find that $L \propto V^{3.4^{+1.3}_{-0.9}}$, whereas they find a different relation: $L \sim V^{0.9}$. We state our conclusions in §5.

2. The Variability Measure

We now rigorously construct a measure of the variability of a burst light curve. We require it to have the following properties: (1) we define it in terms of physical, source-frame quantities, as opposed to measured, observer-frame quantities; (2) when converted to observer-frame quantities, all strong dependences on redshift and other difficult or impossible to measure quantities cancel out; (3) it is not biased by instrumental binning of the light curve, despite cosmological time dilation and the narrowing of the light curve’s temporal substructure at higher energies (Fenimore et al. 1995); (4) it is not biased by Poisson noise, and consequently can be applied to faint bursts; and (5) it is robust; i.e., similar light curves always yield similar variabilities. We also derive an expression for the statistical uncertainty in a light curve’s measured variability. Finally, we

describe how we combine variability measurements of light curves acquired in different energy bands into a single measurement of a burst’s variability.

We first define the variability of a burst light curve, acquired in observer-frame energy band E , in terms of physical, source-frame quantities:

$$V_f^E = \frac{\int_{-\infty}^{\infty} [L^{E_s}(t_s) - (L^{E_s} \star S_f)(t_s)]^2 dt_s}{\int_{-\infty}^{\infty} [L^{E_s}(t_s)]^2 dt_s}, \quad (1)$$

where $L^{E_s}(t_s)$ is the luminosity of the burst in source-frame energy band $E_s = E(1+z)$ as a function of source-frame time t_s , and $(L^{E_s} \star S_f)(t_s)$ is the convolution of this function and a boxcar smoothing function of area equal to 1 and width equal to the smoothing timescale of $L^{E_s}(t_s)$, i.e., the source-frame smoothing timescale of the light curve. Since our definition of the smoothing timescale is a robust measure of duration (§1), Equation (1) is a robust measure of variability.

Next, we convert Equation (1) to observer-frame quantities:

$$V_{f,P}^E = \frac{\int_{-\infty}^{\infty} \{C^E(t_o) - B^E(t_o) - [(C^E - B^E) \star S_f](t_o)\}^2 [\Delta\Omega D^2(z)(1+z)^\alpha]^2 (1+z)^\beta dt_o}{\int_{-\infty}^{\infty} [C^E(t_o) - B^E(t_o)]^2 [\Delta\Omega D^2(z)(1+z)^\alpha]^2 (1+z)^\beta dt_o}. \quad (2)$$

Here, $C^E(t_o)$ and $B^E(t_o)$ are the total (source plus background) and background photon count rates in observer-frame energy band E as a function of observer-frame time t_o , $[(C^E - B^E) \star S_f](t_o)$ is the convolution of $C^E(t_o) - B^E(t_o)$ and a boxcar smoothing function of area equal to 1 and width equal to the smoothing timescale of $C^E(t_o) - B^E(t_o)$, i.e., T_f^E , $\Delta\Omega$ is the effective solid angle of the emitted light, $D(z)$ is the comoving distance to the burst at redshift z , α is either 1 or 2 depending on whether $L^{E_s}(t_s)$ is a photon number or energy luminosity, and $\beta \approx 1 - 0.4 = 0.6$: the factor of $(1+z)^1$ is due to cosmological time dilation, and the factor of $\approx (1+z)^{-0.4}$ is due to the narrowing of the light curve’s temporal substructure at higher energies (Fenimore et al. 1995). By construction, the strong dependences on $\Delta\Omega$, z , α , and β , and a weak dependence on the cosmological model, cancel out. Furthermore, since $B^E(t_o)$ always varies over much longer timescales than T_f^E , Equation (2) simplifies to:

$$V_{f,P}^E = \frac{\int_{-\infty}^{\infty} [C^E(t_o) - (C^E \star S_f)(t_o)]^2 dt_o}{\int_{-\infty}^{\infty} [C^E(t_o) - B^E(t_o)]^2 dt_o}. \quad (3)$$

We distinguish Equations (2) and (3) from Equation (1) with a subscript P because they differ in one fundamental way: unlike $L^{E_s}(t_s)$ in Equation (1), $C^E(t_o)$ suffers from Poisson noise. This additional source of variability biases Equations (2) and (3) to higher values. Although this bias is minor for bursts that are significantly brighter than the background, it cannot be ignored for faint bursts. We determine and subtract out the contribution to the variability due to Poisson noise below.

However, before we can subtract out the contribution to the variability due to Poisson noise, we must first bin the light curve, since observed light curves typically are discrete, not continuous.

Consequently, we replace the integrals in Equation (3) with sums from $i = 1$ to N , $C^E(t_o)$ with C_i^E , $B^E(t_o)$ with B_i^E , and $(C^E \star S_f)(t_o)$ with its discrete equivalent, $S_i(C_j^E, N_f^E)$. Here, C_j^E is the light curve to be smoothed, N_f^E is the smoothing timescale, i.e., T_f^E , but measured in number of bins, and

$$S_i(C_j^E, N_x) = \frac{1}{N_x} \left[\sum_{j=i-n_x}^{i+n_x} C_j^E + \left(\frac{N_x - 1}{2} - n_x \right) (C_{i-n_x-1}^E + C_{i+n_x+1}^E) \right], \quad (4)$$

where n_x is the truncated integer value of $(N_x - 1)/2$. Binning can bias the variability measure, since binning wipes out all variability on timescales shorter than the sampling timescale of the light curve, and the effective sampling timescale of the light curve in the source frame decreases with increasing burst redshift: although the sampling timescale of the light curve is fixed by the instrument in the observer frame, (1) cosmological time dilation and (2) the narrowing of the light curve's temporal substructure at higher energies (Fenimore et al. 1995) decrease the effective sampling timescale of the light curve in the source frame. We remove this redshift bias by smoothing all light curves on a source-frame timescale of 64 ms (the shortest sampling timescale of BATSE), which corresponds to a timescale of $64(1+z)^\beta$ ms in the observer frame. Consequently, we replace C_i^E with $S_i(C_j^E, N_z)$, where $N_z = 64(1+z)^\beta / \Delta t$, and Δt is the observer-frame sampling timescale of the light curve in milliseconds. Equation (3) becomes:

$$V_{f,P}^E = \frac{\sum_{i=1}^N [S_i(C_j^E, N_z) - S_i(C_j^E, N_f^E)]^2}{\sum_{i=1}^N [S_i(C_j^E, N_z) - B_i^E]^2}, \quad (5)$$

where we take C_j^E and B_i^E to be measured in counts, not counts per unit time. Although the computation of $V_{f,P}^E$ now depends on z and β , these dependences are very weak: wide variations in the values of these parameters do not significantly change the measured variabilities of the bursts in our sample; i.e., burst light curves have very little power on such short timescales. Likewise, wide variations in the value of the effective sampling timescale of the light curve that we impose in the source frame also have little effect on the measured variabilities of these bursts.

Now, we determine and subtract out the contribution of Poisson noise to the variability, and simultaneously we derive an expression for the statistical uncertainty in a light curve's measured variability. First, we rewrite the expressions $S_i(C_j^E, N_z) - S_i(C_j^E, N_f^E)$ and $S_i(C_j^E, N_z) - B_i^E$ from Equation (5) as weighted sums of the statistically independent measurements C_j :

$$V_{f,P}^E = \frac{\sum_{i=1}^N (\sum_{j=1}^N a_{ij} C_j)^2}{\sum_{i=1}^N (\sum_{j=1}^N b_{ij} C_j - B_i)^2}, \quad (6)$$

where a_{ij} and b_{ij} are weights that differ for each burst, but can be computed straightforwardly using Equation (4). Since we are in the Gaussian limit, the uncertainty in each C_j is simply $\pm \sqrt{C_j}$. Since each C_j is statistically independent, their weighted uncertainties can be summed in quadrature. Hence, the sums over j in Equation (6) and their uncertainties are: $\sum_{j=1}^N a_{ij} C_j \pm \sqrt{\sum_{j=1}^N a_{ij}^2 C_j}$ and $\sum_{j=1}^N b_{ij} C_j - B_i \pm \sqrt{\sum_{j=1}^N b_{ij}^2 C_j}$.

Next, in accordance with Equation (6), we square these expressions, which yields expressions consisting of three terms: the square of the original term, a positive term due to Poisson noise, and an uncertainty term. Subtracting out the contributions to Equation (6) due to Poisson noise yields:

$$V_f^E = \frac{\sum_{i=1}^N [(\sum_{j=1}^N a_{ij} C_j)^2 - \sum_{j=1}^N a_{ij}^2 C_j]}{\sum_{i=1}^N [(\sum_{j=1}^N b_{ij} C_j - B_i)^2 - \sum_{j=1}^N b_{ij}^2 C_j]}, \quad (7)$$

where the uncertainty in the i th term of the numerator is $\pm 2(\sum_{j=1}^N a_{ij} C_j) \sqrt{\sum_{j=1}^N a_{ij}^2 C_j}$, and the uncertainty in the i th term of the denominator is $\pm 2(\sum_{j=1}^N b_{ij} C_j - B_i) \sqrt{\sum_{j=1}^N b_{ij}^2 C_j}$. Recognizing that only $\approx N/N_z$ of the terms in the sums over i are statistically independent, since the sums over j correspond to a convolution of the light curve and a boxcar smoothing function of width equal to N_z bins, the uncertainties in the sums over i are a factor of $\approx \sqrt{N_z}$ larger than what would be derived if all of the terms were statistically independent. Hence, the uncertainty in the numerator is $\approx \pm 2\sqrt{N_z \sum_{i=1}^N (\sum_{j=1}^N a_{ij} C_j)^2 \sum_{j=1}^N a_{ij}^2 C_j}$, and the uncertainty in the denominator is $\approx \pm 2\sqrt{N_z \sum_{i=1}^N (\sum_{j=1}^N b_{ij} C_j - B_i)^2 \sum_{j=1}^N b_{ij}^2 C_j}$. These uncertainties can be at most only weakly correlated. Taking them to be independent, we find that the statistical uncertainty in a light curve's measured variability is approximately given by:

$$\sigma_{V_f^E} = V_f^E \sqrt{\frac{4N_z \sum_{i=1}^N (\sum_{j=1}^N a_{ij} C_j)^2 \sum_{j=1}^N a_{ij}^2 C_j}{\{\sum_{i=1}^N [(\sum_{j=1}^N a_{ij} C_j)^2 - \sum_{j=1}^N a_{ij}^2 C_j]\}^2} + \frac{4N_z \sum_{i=1}^N (\sum_{j=1}^N b_{ij} C_j - B_i)^2 \sum_{j=1}^N b_{ij}^2 C_j}{\{\sum_{i=1}^N [(\sum_{j=1}^N b_{ij} C_j - B_i)^2 - \sum_{j=1}^N b_{ij}^2 C_j]\}^2}}. \quad (8)$$

All that remains is to describe how we combine variability measurements of light curves acquired in different energy bands, V_f^E , into a single measurement of a burst's variability: V_f . For each burst in our sample, light curves were acquired in typically 3 or 4 independent energy bands. We find that (1) the smoothing timescales of these bursts' light curves decrease with energy as $\approx E^{-0.4}$ (Figure 3), and (2) the variabilities of these bursts' light curves are approximately constant across energy bands (Figure 4). In hindsight, the former result is not surprising, given our definition of the smoothing timescale and the principle result of Fenimore et al. (1995); however, this does constitute an independent confirmation of their result. The latter result suggests that we can model a burst's variability as a constant across energy bands. Applying the Bayesian inference formalism developed by Reichart (2000) for fitting models with extrinsic scatter (sample variance) to data with intrinsic scatter (statistical variance, in this case given by Equation 8), we fit this model to the typically 3 or 4 independent measurements of a burst's variability made in independent energy bands, V_f^E , resulting in a single measurement of that burst's variability, V_f , and the uncertainty in V_f .

3. The Luminosity Measure

Let P be the peak flux of a burst in photons $\text{cm}^{-2} \text{s}^{-1}$ between observer-frame energies E_l and E_u . The isotropic-equivalent peak luminosity of the burst in erg s^{-1} between source-frame energies 100 and 1000 keV is given by

$$L = 4\pi D^2(z) P \frac{\int_{100}^{1000} E \Phi\left(\frac{E}{1+z}\right) dE}{\int_{E_l}^{E_u} \Phi(E) dE}, \quad (9)$$

where $\Phi(E)$ is the observer-frame spectral shape, which we parameterize with the Band function (Band et al. 1993). For each burst in our sample, we compute the value of L for each of the 54 parameterizations of the Band function in Band et al. (1993); the luminosity error bars in Figures 2 and 3 are dominated by this variation in the parameterization of the Band function, but clearly are too small to matter. Hence, L is very insensitive to reasonable variations in the parameterization of the Band function for the bursts in our sample. The choice of source-frame energy range also is very unimportant: we chose 100 – 1000 keV to approximately match the observer-frame energy range in which BATSE measures peak fluxes, 50 – 300 keV, for redshifts that are typical of the bursts in our sample: $z \approx 1 - 2$.

4. The Luminosity Estimator

We list our sample of 20 bursts in Table 1; it consists of every burst for which redshift information is currently available. Peak fluxes are available for all 20 bursts. Spectroscopic redshifts and 64-ms or better resolution light curves are available for 11 bursts. Spectroscopic redshifts but only 1-s resolution light curves are available for 2 bursts. We compute variability lower limits for these bursts: we compute variabilities from their 1-s resolution light curves without further degrading the effective resolution of these light curves by smoothing them on the 64-ms source-frame timescale (§2); these variabilities are lower limits to the variabilities that we would compute if 64-ms or better resolution light curves were available. Redshift upper limits (1) from the non-detection of the $\text{Ly}\alpha$ forest in host galaxy spectroscopy in the case of 1 burst, and (2) from the non-detection of the Lyman limit in afterglow and host galaxy photometry in the case of 6 bursts, and 64-ms or better resolution light curves are available for the remaining 7 bursts. We compute luminosity upper limits for these bursts. We compute variabilities for these bursts for both $z = 0$ and z equal to the redshift upper limit; for all 7 bursts, both of these variabilities are nearly identical, testifying to the weakness of the computational dependence of the variability on redshift (§2). In addition to the data listed in Table 1, we used *Ulysses*/GRB data for GRBs 970228, 990712, and 991208 to test the consistency of our results across instruments; we find the measured variabilities and luminosities of these bursts to be fully consistent across instruments.

We now construct the luminosity estimator. First, we compute the variabilities, V_f , and the

isotropic-equivalent luminosities, L ,⁷ of the bursts in our sample. We compute the variabilities as functions of f , where we use the value of f to compute the smoothing timescales, T_f^E , of all of the light curves in our sample (§1). We identify values of f that lead to successful and robust luminosity estimators below. Regardless of the value of f that we use to compute the variabilities, the distribution of our sample’s bursts in the $\log L$ - $\log V_f$ plane appears to be well modeled by a normal distribution about a straight line (see, e.g., Figure 7). We parameterize the line by:

$$\log V_f(L) = \log \bar{V}_f + b + m(\log L - \log \bar{L}), \quad (10)$$

where b is the intercept of the line, m is its slope, and \bar{V}_f and \bar{L} are the median values of V_f and L for the 11 bursts in our sample for which spectroscopic redshifts, peak fluxes, and 64-ms or better resolution light curves are available. We parameterize this line as a function of L , instead of as a function of V_f , because the data do not fully rule out the possibility of $m \lesssim 0$, whereas the data do rule out the possibility of very large values of m . We parameterize the normal distribution about this line by $\sigma_{\log V_f}$, which is half of the distribution’s $1\text{-}\sigma$ width along the $\log V_f$ axis. Applying the Bayesian inference formalism developed by Reichart (2000) for fitting data with extrinsic scatter (in this case, $\sigma_{\log V_f}$) to data with intrinsic scatter (in this case, the uncertainties in the measured values of V_f and L), we determine values and uncertainties for the model parameters (b , m , and $\sigma_{\log V_f}$) as functions of f (Figure 5).⁸

Of the 20 bursts in our sample, GRB 980425 is unique due to its possible association with SN 1998bw (Kulkarni et al. 1998b; Galama et al. 1998) at a redshift of $z = 0.0085$ (Tinney et al. 1998); the 12 other bursts for which spectroscopic redshifts are available have $0.430 < z < 3.418$. Consequently, we first construct the luminosity estimator excluding GRB 980425 from the above fits; we then repeat the fits including this burst. Excluding GRB 980425 from the fits, we first identify values of f that lead to successful and robust luminosity estimators. One measure of the success of a luminosity estimator is the probability that its slope, m , departs from $m = 0$. Since the best-fit values of m are positive (Figure 5), we compute and also plot in Figure 5 the probability that $m < 0$, $P(m < 0)$, as a function of f . We find that values of $f \lesssim 0.5$ lead to luminosity estimators with slopes that are positive with approximately equally probability: $P(m < 0) \approx 6.4 \times 10^{-4}$ (3.4σ). Values of $f \gtrsim 0.5$ lead to luminosity estimators with slopes that are positive with increasingly less probability. In addition to having a slope that is positive (or negative) with large probability, a successful luminosity estimator also should be robust: small changes in the value of f should lead to only small changes in the values of the fitted model

⁷We use peak fluxes measured on a 1-s timescale, and we take $H_0 = 65 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_m = 0.3$, and $\Omega_\Lambda = 0.7$; the luminosity estimator is very insensitive to these choices.

⁸The Bayesian inference formalism developed by Reichart (2000) deals only with measurements with Gaussian error distributions, not with lower or upper limits. However, this formalism can be straightforwardly generalized to deal with limits as well, using two facts: (1) that a limit can be given by the convolution of a Gaussian distribution and a Heavyside function, and (2) that convolution is associative. In this paper, we fit to both measurements and limits.

parameters. From Figure 5, we see that values of $f \gtrsim 0.3 - 0.4$ lead to very robust luminosity estimators; lower values of f lead to increasingly less robust luminosity estimators. Taking $f = 0.45$, which corresponds to a local minimum in $P(m < 0)$, we find that $\log \bar{V}_f = -0.977$, $\log \bar{L} = 52.153$, $b = -0.062_{-0.095}^{+0.088}$, $m = 0.290_{-0.098}^{+0.112}$, and $\sigma_{\log V_f} = 0.176_{-0.044}^{+0.077}$. We plot the two-dimensional uncertainty distributions of the fitted values of the model parameters in Figure 6. We plot the data and the best-fit model of the distribution of these data in the $\log L$ - $\log V_f$ plane for $f = 0.45$ in Figure 7.

Although the solid line in Figure 7 is a good approximation to the best estimate for L as a function of V_f , the dotted lines in Figure 7 do not correspond to the uncertainty in L as a function of V_f . The dotted lines in Figure 7 mark the best-fit width, given by $\sigma_{\log V_f} = 0.176$, of the best-fit model of the distribution of the data in the $\log L$ - $\log V_f$ plane; they do not account for the uncertainties in the fitted values of the model parameters, and the uncertainties in the fitted values of b and m in particular. Applying the Bayesian inference formalism of Reichart (2000), we formally compute the best estimate for L as a function of V_f , and the uncertainty in L as a function of V_f , which we plot as the solid curves in Figure 8. The computation of this distribution in the $\log L$ - $\log V_f$ plane is numerically intensive, so we also provide analytic approximations to these functions, which we plot as the dotted curves in Figure 8. The best estimate for L as a function of V_f can be approximated by Equation (10), using the best-fit values of the model parameters. Approximating the uncertainty distributions of the fitted values of the model parameters (Figure 6) as uncorrelated and Gaussian, the uncertainty in L as a function of V_f can be approximated by (Reichart 2000):

$$\sigma_{\log V_f}(L) = \sqrt{\sigma_{\log V_f}^2 + \sigma_b^2 + \sigma_m^2(\log L - \log \bar{L})^2}, \quad (11)$$

where $\sigma_{\log V_f}(L)$ is the 1- σ uncertainty in V_f as a function of L , $\sigma_{\log V_f} = 0.176$, $\sigma_b \approx 0.092$ is the uncertainty in the fitted value of b , and $\sigma_m \approx 0.105$ is the uncertainty in the fitted value of m . At its best, the luminosity estimator constructed excluding GRB 980425 from the fits yields best-estimate luminosities that are accurate to a factor of ≈ 5 , or best-estimate luminosity distances that are accurate to a factor of ≈ 2 .

In addition to being used to estimate the luminosities of bursts for which spectroscopic redshifts are not available, and the uncertainties in these luminosities, Figure 8 also can be used to determine if other bursts for which spectroscopic redshifts are available, such as GRB 980425 or future bursts, are consistent with the above-fitted model. Consequently, we also plot GRB 980425 in Figure 8. Although its implied luminosity, $L = 1.0 \pm 0.4 \times 10^{47}$ erg s $^{-1}$, is $\approx 4 - 6$ orders of magnitude less than the 12 other measured luminosities of bursts in our sample, its variability, $V_{f=0.45} = 0.00 \pm 0.01$, also is considerably less than the ≈ 16 other reasonably well-constrained variabilities of bursts in our sample, and clearly is consistent with the above-fitted model. Repeating the fits including GRB 980425 yields: $f = 0.47$, $\log \bar{V}_f = -1.069$, $\log \bar{L} = 51.998$, $b = -0.006_{-0.100}^{+0.087}$, $m = 0.291_{-0.078}^{+0.108}$, $\sigma_{\log V_f} = 0.173_{-0.044}^{+0.078}$, and $P(m < 0) = 2.1 \times 10^{-6}$ (4.7 σ) (Figure 5). We also plot the two-dimensional uncertainty distributions of the fitted values of the model parameters in Figure 6. We plot the data and the best-fit model of the distribution of

these data in the $\log L$ - $\log V_f$ plane for $f = 0.47$ in Figure 9. We plot the best estimate for L as a function of V_f , the uncertainty in L as a function of V_f , and our analytic approximations to these functions, given by Equations (10) and (11) and the fitted values of the model parameters, in Figure 10.

5. Discussion and Conclusions

We have presented a rigorously-constructed measure of the variability of a burst’s light curve. Using this variability measure and a sample of 20 bursts, consisting of every burst for which redshift information is currently available, we have constructed a possible Cepheid-like luminosity estimator for long bursts. If one excludes GRB 980425 from the fits, the luminosity estimator spans ≈ 2.5 orders of magnitude in luminosity, and its slope is positive with a probability of $1 - 6.4 \times 10^{-4}$ (3.4σ). GRB 980425, however, is consistent with the fitted model. If one includes this burst in the fits, the luminosity estimator spans ≈ 6.3 orders of magnitude in luminosity, and its slope is positive with a probability of $1 - 2.1 \times 10^{-6}$ (4.7σ).

Future bursts will either increase these probabilities, or possibly disprove the luminosity estimator. However, since the uncertainty in L as a function of V_f in Figures 8 and 10 is dominated by extrinsic scatter (i.e., $\sigma_{\log V_f}$), and not by the uncertainties in the fitted values of the model parameters, a larger sample of bursts is unlikely to improve the predictive power of the luminosity estimator. Currently, the luminosity estimator yields best-estimate luminosities that are accurate to a factor of ≈ 5 , or best-estimate luminosity distances that are accurate to a factor of ≈ 2 .

However, independently of whether or not the luminosity estimator is eventually disproved, the light curve of GRB 980425 is unique in that it is much less variable than the other ≈ 16 light curves of bursts in our sample for which the signal-to-noise is reasonably good: $\log V_{f=0.47} < -1.5$ with a probability of $1 - 8.7 \times 10^{-4}$ (3.3σ), and $\log V_{f=0.47} < -1$ with a probability of $1 - 7.4 \times 10^{-11}$ (9.7σ). The argument has been made that the association of GRB 980425 with SN 1998bw at the unusually low redshift of $z = 0.0085$ probably is accidental because the light curve of GRB 980425 is no different than the light curves of the cosmological bursts. On the contrary, we find that the light curve of GRB 980425 is different from the light curves of the cosmological bursts. Consequently, GRB 980425 may well be associated with SN 1998bw.

If GRB 980425 is associated with SN 1998bw, and if the luminosity estimator is correct, the fact that GRB 980425 is consistent with the fitted model suggests that GRB 980425 and the cosmological bursts may share a common, or at least a related, physical origin, although they cannot share a common redshift distribution and/or luminosity function (Graziani, Lamb, & Marion 1999). This conclusion is made more intriguing by the recent discoveries of supernova-like components to the late afterglows of the cosmological bursts GRB 970228 (Reichart 1999, Galama et al. 2000; Reichart, Castander, & Lamb 2000; Reichart, Lamb, & Castander 2000) and GRB 980326 (Bloom et al. 1999).

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Table 1. GRB Redshift Sample

GRB	z	L^a	$V_{f=0.47}$	Instrument	z Reference
970228	0.695	$1.0 \pm 0.3 \times 10^{52}$	0.12 ± 0.10	<i>Wind</i> /KONUS	Djorgovski et al. 1999b
970508	0.835	$2.0 \pm 0.5 \times 10^{51}$	0.08 ± 0.03	<i>CGRO</i> /BATSE	Metzger et al. 1997
970828	0.958	$1.4 \pm 0.3 \times 10^{52}$	0.11 ± 0.00	<i>CGRO</i> /BATSE	Djorgovski 1999
971214	3.418	$1.2 \pm 0.3 \times 10^{53}$	0.12 ± 0.01	<i>CGRO</i> /BATSE	Kulkarni et al. 1998a
980308	$< 4.3^b$	$< 1.5 \times 10^{53}$	0.25 ± 0.04	<i>CGRO</i> /BATSE	Schaefer et al. 1999
980326	$< 4.3^b$	$< 2.5 \times 10^{53}$	0.12 ± 0.02	<i>CGRO</i> /BATSE	Groot et al. 1998
980329	$< 3.9^c$	$< 1.1 \times 10^{54}$	0.04 ± 0.01	<i>CGRO</i> /BATSE	Djorgovski 1999
980425	0.0085	$1.0 \pm 0.4 \times 10^{47}$	0.00 ± 0.01	<i>CGRO</i> /BATSE	Tinney et al. 1998
980519	$< 4.3^b$	$< 4.4 \times 10^{54}$	0.17 ± 0.03	<i>CGRO</i> /BATSE	Halpern et al. 1999
980613	1.096	$2.6 \pm 0.5 \times 10^{51}$	$> 0.04^d$	<i>CGRO</i> /BATSE	Djorgovski et al. 1999a
980703	0.967	$7.1 \pm 1.4 \times 10^{51}$	0.06 ± 0.01	<i>CGRO</i> /BATSE	Djorgovski et al. 1998
981220	$< 4.3^b$	$< 8.7 \times 10^{53}$	0.09 ± 0.06	<i>Wind</i> /KONUS	Masetti et al. 1998
990123	1.600	$1.7 \pm 0.2 \times 10^{53}$	0.11 ± 0.01	<i>CGRO</i> /BATSE	Kulkarni et al. 1999
990510	1.619	$8.7 \pm 1.0 \times 10^{52}$	0.26 ± 0.01	<i>CGRO</i> /BATSE	Beuermann et al. 1999
990705	$< 5.5^b$	$< 3.5 \times 10^{53}$	0.13 ± 0.22	<i>Ulysses</i> /GRB	Masetti et al. 2000
990712	0.430	$7.6 \pm 3.0 \times 10^{50}$	0.03 ± 0.04	<i>Wind</i> /KONUS	Galama et al. 1999
991208	0.706	$5.7 \pm 1.9 \times 10^{52}$	0.09 ± 0.04	<i>Wind</i> /KONUS	Djorgovski et al. 1999c
991216	1.020	$2.3 \pm 0.4 \times 10^{53}$	0.17 ± 0.02	<i>CGRO</i> /BATSE	Vreeswijk et al. 1999
000131	$< 5.5^b$	$< 1.3 \times 10^{54}$	0.12 ± 0.02	<i>Wind</i> /KONUS	Pedersen et al. 2000
000301	2.034	$1.2 \pm 0.4 \times 10^{53}$	$> 0.03^d$	NEAR/XGRS ^e	Castro et al. 2000

^aIsotropic-equivalent luminosity in erg s^{-1} between source-frame energies 100 and 1000 keV, for peak fluxes measured on a 1-s timescale, $H_0 = 65 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_m = 0.3$, and $\Omega_\Lambda = 0.7$; upper limits are 1σ .

^bFrom non-detection of Lyman limit in afterglow or host galaxy photometry.

^cFrom non-detection of Ly α forest in host galaxy spectroscopy.

^dOnly 1-s resolution light curve is available; lower limit is 1σ .

^ePeak flux from *Ulysses*/GRB.

Fig. 1.— A schematic illustration of our measure of burst duration. The solid line represents a burst light curve, acquired in observer-frame energy band E , and the dashed line represents the background. Top panel: The hashed area is the brightest 50% of the total counts above background. Hence, the 50% duration, or smoothing timescale, of this light curve is given by $T_{f=0.5}^E = T_1 + T_2$. Bottom panel: The hashed area is the brightest 90% of the total counts above background. Hence, the 90% duration, or smoothing timescale, of this light curve is given by $T_{f=0.9}^E = T_3$.

Fig. 2.— Another schematic illustration of our measure of burst duration. The solid line represents a burst light curve, acquired in observer-frame energy band E , and the dashed line represents the background. Top panel: The hashed area is the brightest 50% of the total counts above background. Hence, the 50% duration, or smoothing timescale, of this light curve is given by $T_{f=0.5}^E = T_4$. Bottom panel: The hashed area is the brightest 90% of the total counts above background. Hence, the 90% duration, or smoothing timescale, of this light curve is given by $T_{f=0.9}^E = T_5 + T_6$.

Fig. 3.— Bursts in our sample for which spectroscopic redshifts, peak fluxes, and 64-ms or better resolution light curves are available. For each of these bursts, light curves were acquired in typically 3 or 4 independent energy bands E . For each burst, we compute the durations, or smoothing timescales, $T_{f=0.47}^E$ of its 3 or 4 independent light curves, and plot $T_{f=0.47}^E$ as a function of source-frame energy band $E_s = E(1+z)$. The horizontal bars mark the source-frame energy bands. We plot solid lines if the light curve has > 3000 total counts above background, and dotted lines if the light curve has < 3000 total counts above background. The dotted portions of these curves suffer from high S/N; very little variability information can be gleaned from these light curves. However, the trend is clear: the durations, or smoothing timescales, $T_{f=0.47}^E$ of these bursts are shorter at higher energies. From the solid portions of these curves, we find that $T_{f=0.47}^E \sim E^{-0.4}$, in agreement with the principle result of Fenimore et al. (1995). Other values of f yield similar results.

Fig. 4.— Bursts in our sample for which spectroscopic redshifts, peak fluxes, and 64-ms or better resolution light curves are available. For each of these bursts, light curves were acquired in typically 3 or 4 independent energy bands E . For each burst, we compute the variabilities $V_{f=0.47}^E$ of its 3 or 4 independent light curves, and the combined variability $V_{f=0.47}$ of the burst. We plot $\log V_{f=0.47}^E - \log V_{f=0.47}$ versus source-frame energy band $E_s = E(1+z)$. The horizontal bars mark the source-frame energy bands. Clearly, there is no significant trend with energy. Other values of f yield similar results.

Fig. 5.— The value of $\log \bar{V}_f$ as a function of f , the best-fit values, and 1σ uncertainties in the best-fit values, of the model parameters b , m , and $\sigma_{\log V_f}$ as functions of f , and the probability $P(m < 0)$ that $m < 0$ as a function of f . In the left panels, we exclude GRB 980425 from the fits; in the right panels we include GRB 980425 in the fits.

Fig. 6.— The two-dimensional uncertainty distributions of the fitted values of the model parameters b , m , and $\sigma_{\log V_f}$. The curves mark the 1σ credible regions. The dotted curves are for the case in which we exclude GRB 980425 from the fits ($f = 0.45$); the solid curves are for the case in which

we include GRB 980425 in the fits ($f = 0.47$).

Fig. 7.— The variabilities $V_{f=0.45}$ and isotropic-equivalent luminosities L of the bursts in our sample, excluding GRB 980425. The solid and dotted lines mark the center and 1σ widths of the best-fit model distribution of these bursts in the $\log L$ - $\log V_{f=0.45}$ plane.

Fig. 8.— The solid lines mark the best estimate for L as a function of $V_{f=0.45}$, and the 1σ uncertainty in L as a function of $V_{f=0.45}$ for the case in which we exclude GRB 980425 from the fits. The dotted lines mark analytic approximations to these functions, given by Equations (10) and (11) and the fitted values of the model parameters. Clearly, GRB 980425 is consistent with the fitted model.

Fig. 9.— The variabilities $V_{f=0.47}$ and isotropic-equivalent luminosities L of the bursts in our sample, including GRB 980425. The solid and dotted lines mark the center and 1σ widths of the best-fit model distribution of these bursts in the $\log L$ - $\log V_{f=0.47}$ plane.

Fig. 10.— The solid lines mark the best estimate for L as a function of $V_{f=0.47}$, and the 1σ uncertainty in L as a function of $V_{f=0.47}$ for the case in which we include GRB 980425 in the fits. The dotted lines mark analytic approximations to these functions, given by Equations (10) and (11) and the fitted values of the model parameters.



















